Eur. Phys. J. C 8, 465–471 (1999)
DOI 10.1007/s100529901071

THE EUROPEAN
PHYSICAL JOURNAL C

The masses of the 0^{++} and 0^{-+} light-quark hybrids using QCD sum rules

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Received: 17 July 1998 / Revised version: 11 September 1998 / Published online: 15 April 1999

Abstract. We calculate the masses of the light-quark hybrid mesons with the quantum numbers 0^{++} and 0^{-+} by using the QCD sum rules. Two kinds of interpolated currents with the same quantum numbers are employed. We find that approximately equal masses are predicted for the 0^{-+} hybrid state using the different currents, and different masses are obtained for the 0^{++} hybrid state using the different currents. The prediction depends on the interaction between the gluon and the quarks in the low-lying hybrid mesons. The mixing effect on the mass of the light-quark hybrid mesons through the low-energy theorem has also been examined, and it is found that this mixing shifts the masses of the hybrid mesons and the glueball a little.

1 Introduction

It is believed that the gluon degrees of freedom play an important role in hadrons. QCD theory predicts the existence of a glueball and of hybrid states. Searches for glueballs and hybrid mesons in experiments has been carried out for a long time since the 1980's; so far there is no conclusive evidence of them [1]. Glueballs and hybrids are particularly difficult to identify in experiments since their mass spectroscopy overlaps with the ordinary $\bar{q}q$ meson spectroscopy and they can mix with each other. From a theoretical point of view, glueballs and hybrids have been discussed in terms of the bag model [2], the potential model [3], the flux-tube model [4] and the QCD sum rules [5–7], but we have no effective non-perturbative theory in QCD to predict their masses precisely.

The calculation of hybrid mesons using QCD sum rules [8] was first given by Balitsky *et al.* [5]; then two other groups, i.e. Latorre *et al.* [6] and Govaerts *et al.* [7], independently gave a revised calculation on the hybrid mesons. There are some errors in their previous calculation, but they corrected these later. To avoid dealing with the mixing of ordinary mesons with the hybrids, the former two groups focused attention only on the hybrids with exotic quantum numbers ($J^{pc} = 1^{-+}, 0^{--}$). They obtained the masses and decay amplitudes of these hybrids. The third group performed a mass calculation not only for the $J^{pc} = 1^{-+}, 0^{--}$ hybrids, but also for the $J^{pc} = 1^{+-}, 0^{++}$ hybrids. The decay widths for some decay modes of the 1^{-+} hybrid [9] were calculated by the latter two groups too.

For the heavy-quark hybrids, Govaerts *et al.* presented the mass calculation with various J^{pc}. They analyzed sets of coupled sum rules by using the different interpolated currents and found that the mass predictions for the same J^{pc} from totally different sum rules essentially agree with each other within the errors of their procedure. States with the same J^{pc} were considered to be identical. For light-quark hybrids, all predictions of these three groups of the mass of the exotic 1^{-+} hybrid agree with the recent experimental result of [1]. For the normal hybrids ($J^{pc} = 1^{--}, 0^{++}$), they did not consider the mixing effect of hybrids with ordinary mesons which is supposed to exist. The whole calculation results only from the vector current $g\bar{q}\gamma_\alpha G^a_{\alpha\mu} T^a q(x)$.

In this paper, we first extend the approach of Govaerts *et al.* to the light-quark case for the 0^{++} and 0^{-+} hybrids by using two kinds of interpolated currents: $g\bar{q}\sigma_{\mu\nu} G^a_{\nu\mu} T^a q(x)$ and $g\bar{q}\gamma_\alpha G^a_{\alpha\mu} T^a q(x)$. The $\bar{q}q$ combination in the current $g\bar{q}\sigma_{\mu\nu} G^a_{\nu\mu} T^a q(x)$ can be considered to have the quantum numbers $J^{pc} = 1^{+-}$ and the $\bar{q}q$ combination in the current $g\bar{q}\gamma_\alpha G^a_{\alpha\mu} T^a q(x)$ has $J^{pc} = 1^{--}$; the interaction between quarks and gluon in these two currents differs accordingly. Thus, one cannot expect the same mass prediction from these two different currents using the light-quark hybrid mesons sum rules. This is similar to the situation of hybrid mesons in the MIT bag model [2]. For instance, the $\bar{q}q$ combination of the 0^{++} hybrid meson $\bar{q}qg$ may have $J^{pc} = 1^{--}$ with the gluon in the TE(1^{--}) mode [10] or $J^{pc} = 1^{+-}$ with the gluon in the TM(1^{+-}) mode. These two 0^{++} states have a different intrinsic structure and energy. Therefore, the hybrid mesons with the same J^{pc} can be obtained from totally different sum rules by using different interpolated currents. We calculate the masses of the light-quark hybrid mesons, the 0^{++} and 0^{-+} states, by using two different currents. Our result shows that the prediction depends on the interaction between quarks and gluon in the low-lying hybrid meson. An approximately equal mass is predicted for the 0^{-+} hybrid mesons from the different currents, and different masses are obtained for the 0^{++} hybrid mesons from the two different currents.

Secondly, we consider the mixing effect on the mass determination of the hybrid mesons between the low-lying 0^{++} glueball and the hybrid meson $\bar{q}qg$. By using the low-energy theorem [11], we can construct a sum rule for the mixing correlation function (one gluonic current and one hybrid current). Through this relationship and based on the assumption of the dominance of the two states (lowest-lying states of glueball and hybrid meson $\bar{q}qg$), we find that the mass for the 0^{++} glueball is around 1.8 GeV, which is a little higher than the pure-resonance prediction, and the mass for the 0^{++} $\bar{q}qg$ hybrid meson is around 2.6 GeV, which is also a little higher than the pure-resonance prediction.

This paper is organized as follows. The analytic formalism of the QCD sum rules for the hybrids is given in Sect. 2. In Sect. 3 we give the numerical results for the masses of the 0^{++} and 0^{-+} light-quark hybrid mesons and compare these with those in the bag model with the same J^{pc}. The mixing effect of the glueball with the hybrid meson state is studied in Sect. 4. The last section contains a summary.

2 QCD sum rules for light-quark hybrid mesons

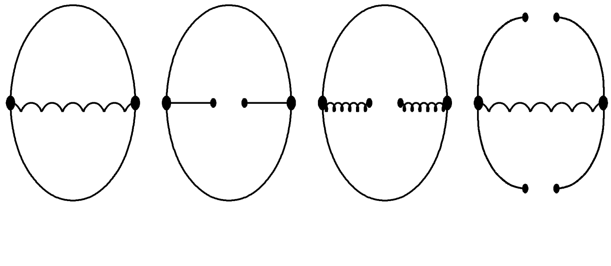
To construct the sum rules for the light-quark hybrid mesons $\bar{q}qg$, we use composite operators with the same quantum numbers as these states to build the correlation functions. In order to obtain the 0^{++} hybrid meson sum rules, we define two different currents

$$j(x) = g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a q(x), \quad (1)$$
$$j_\mu(x) = g\bar{q}\gamma_\alpha G^a_{\alpha\mu}T^a q(x), \quad (2)$$

where $q(x)$ and $G^a_{\alpha\mu}(x)$ are the light-quark field and the gluon field-strength tensor, respectively. T^a are the color matrices.

Through the OPE, we expand the correlation function of $j(x)$ in the background field gauge [12] only in the leading order, which includes the perturbative part (a), the two-quark condensate (b), the two-gluon condensate (c) and the four-quark condensate (d). The result can be obtained from the Feynman diagrams in Fig. 1a–d:

$$\begin{aligned}\Pi(q^2) &= \mathrm{i}\int \mathrm{e}^{\mathrm{i}qx}\langle 0|T\{j(x),j(0)\}|0\rangle\,\mathrm{d}x \quad (3)\\ &= -A(q^2)^3\ln(-q^2/\Lambda^2) - Bq^2\ln(-q^2/\Lambda^2)\\ &\quad -C\ln(-q^2/\Lambda^2) - D\frac{1}{q^2} + \text{const.},\end{aligned}$$



(a) **(b)** **(c)** **(d)**

Fig. 1. Feynman diagrams of the leading order contributing to the correlation function

where

$$A = \frac{\alpha_s}{24\pi^3},\ \ B = \frac{4\alpha_s}{\pi}\langle m\bar{q}q\rangle,$$
$$C = -\frac{m^2}{\pi}\langle\alpha_s G^2\rangle,\ \ D = \frac{8\pi\alpha_s}{3}\langle m\bar{q}q\rangle^2.$$

When the u and d quarks are taken to be massless, the coefficient C vanishes.

In order to relate the QCD calculation with hadron physics, the standard dispersion relation

$$\Pi(q^2) = \frac{1}{\pi}\int\frac{\mathrm{Im}\,\Pi(s)}{s-q^2}\,\mathrm{d}s \quad (4)$$

is used; the spectral density $\mathrm{Im}\,\Pi(s)$ is satisfied by a single narrow resonance and a continuum in a θ-function form. It is given by the following expression:

$$\mathrm{Im}\,\Pi(s) = \pi g_R^2(m_R^2)^4\delta(s-m_R^2)+\pi(As^3+Bs+C)\theta(s-s_0), \quad (5)$$

where g_R is the coupling of the current to the hybrid meson state and m_R refers to the mass of the hybrid meson.

In practice, to proceed with the sum-rule calculation, it is more convenient to define the moments R_k [14], which are expressed by

$$\begin{aligned}R_k(\tau,s_0) &= \frac{1}{\tau}\hat{L}\left[(q^2)^k\left\{\Pi(Q^2)-\sum_{k=0}^{n-1}a_k(q^2)^k\right\}\right] \quad (6)\\ &\quad -\frac{1}{\pi}\int_{s_0}^{+\infty}s^k\mathrm{e}^{-s\tau}\mathrm{Im}\,\Pi^{\{\mathrm{pert}\}}(s)\,\mathrm{d}s\\ &= \frac{1}{\pi}\int_0^{s_0}s^k\mathrm{e}^{-s\tau}\mathrm{Im}\,\Pi(s)\,\mathrm{d}s,\end{aligned}$$

where $\hat{L}$ is the Borel transformation, τ is the Borel transformation parameter, s_0 is the starting point of the continuum threshold, and $\sum_{k=0}^{n-1}a_k(q^2)^k$ are some subtraction constants.

By substituting (3) into (6), it is found that $R_0(\tau,s_0)$ behaves as

$$\begin{aligned}R_0(\tau,s_0) &= \frac{1}{\tau^4}\{6A[1-\rho_3(s_0\tau)] + B\tau^2[1-\rho_1(s_0\tau)]\\ &\quad +C\tau^3[1-\rho_0(s_0\tau)] + D\tau^4\}, \quad (7)\end{aligned}$$

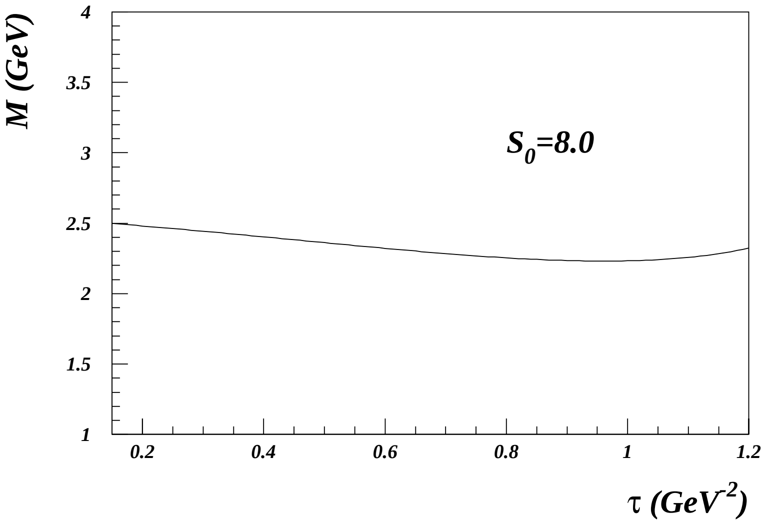


Fig. 2. 0^{++} $\bar{s}sg$ mass from R_1/R_0 versus τ at $s_0 = 8.0\,\mathrm{GeV}^2$ corresponding to the current $j(x)$

where

$$\rho_k(x) = \mathrm{e}^{-x}\sum_{j=0}^{k}\frac{x^j}{j!} \qquad (8)$$

and higher moments R_k can be related to R_0:

$$R_k(\tau, s_0) = \left(-\frac{\partial}{\partial\tau}\right)^k R_0(\tau, s_0). \qquad (9)$$

As done for (3), we can calculate the correlator $\Pi_{\mu\nu}(q^2)$ from the current $j_\mu(x)$:

$$\begin{aligned}\Pi_{\mu\nu}(q^2) &= \mathrm{i}\int \mathrm{e}^{\mathrm{i}qx}\langle 0|T\{j_\mu(x), j_\nu(0)\}|0\rangle\,\mathrm{d}x \qquad (10)\\ &= \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\Pi_v(q^2) + \frac{q_\mu q_\nu}{q^2}\Pi_s(q^2),\end{aligned}$$

and

$$\begin{aligned}\Pi_s(q^2) &= -A'(q^2)^3\ln(-q^2/\Lambda^2) - B'q^2\ln(-q^2/\Lambda^2)\\ &- \quad C'\ln(-q^2/\Lambda^2) - D'\frac{1}{q^2} + const., \qquad (11)\end{aligned}$$

where

$$A' = \frac{\alpha_\mathrm{s}}{480\pi^3},\ B' = -\left(\frac{\alpha_\mathrm{s}}{3\pi}\langle m\bar{q}q\rangle + \frac{\langle\alpha_\mathrm{s}G^2\rangle}{24\pi}\right),$$
$$C' = -\frac{m^2}{8\pi}\langle\alpha_\mathrm{s}G^2\rangle,\ D' = -\frac{2\pi\alpha_\mathrm{s}}{3}\langle m\bar{q}q\rangle^2;$$

the coefficient of the two-quark condensate in B' is a little different from [7]. The $\Pi_v(q^2)$ is the same as in this reference.

Replacing the $G^a_{\alpha\mu}(x)$ in (1) and (2) by

$$\tilde{G}^a_{\alpha\mu}(x) = \frac{1}{2}\epsilon_{\alpha\mu\rho\sigma}G^a_{\rho\sigma}(x), \qquad (12)$$

we arrive at the sum rules for the resonance states with opposite parity (0^{-+}). The results of the correlation functions and moments are almost the same as before, except that the sign of the gluon condensate is changed.

3 Numerical results and J^{pc} analysis

From (6), the mass of the hybrid meson is given by (with $k \geq 0$)

$$m_R^2 = \frac{R_{k+1}}{R_k}; \qquad (13)$$

the moments R_1/R_0 and R_2/R_1 are both suitable for the mass determination according to the ordinary QCD sum rules criteria. They are employed in the following calculation.

To get the numerical results, the parameters are chosen to be

$$\Lambda = 0.2\,\mathrm{GeV},\ m_s = 0.15\,\mathrm{GeV},$$
$$\langle\bar{q}q\rangle = -(0.25\,\mathrm{GeV})^3,\ \langle m\bar{q}q\rangle = -(0.1\,\mathrm{GeV})^4,$$
$$\langle m\bar{s}s\rangle = -0.15*0.8*0.25^3\,\mathrm{GeV}^4,\ \frac{\langle\alpha_\mathrm{s}G^2\rangle}{\pi} = 0.33^4\,\mathrm{GeV}^4,$$
$$\alpha_\mathrm{s}(\tau) = -\frac{4\pi}{9\ln(\tau\Lambda^2)},$$

where q refers to the u or d quark field.

Corresponding to the current $j(x)$ in (1), the mass of the 0^{++} $\bar{s}sg$ hybrid meson, determined from R_1/R_0, is shown in Fig. 2 and one finds 2.35 GeV. If we use R_2/R_1, the result is almost the same, ~ 2.30 GeV. When the quark mass vanishes, which corresponds to $q = u, d$, the result changes a little.

Corresponding to the current $j_\mu(x)$ in (2), R_1/R_0 gives the mass of the 0^{++} $\bar{s}sg$ hybrid meson a value around 3.4 GeV (corresponding to the dotted line in Fig. 3) and the higher moment shifts the mass a little lower. When the quark mass goes to zero, the mass shifts a little compared to the strange-quark case.

There is no plateau in the case of the 0^{-+} hybrid meson. We deal with it as in [7]. The masses of the 0^{-+} hybrid mesons corresponding to the currents $j(x)$ and $j_\mu(x)$ have an approximately equal value: 2.3 GeV. They are shown in Fig. 4.

All of these results are obtained at a suitable s_0, which accounts for the ordinary QCD sum-rule criteria for choosing the threshold. The s_0 for the current $j(x)$ is chosen to

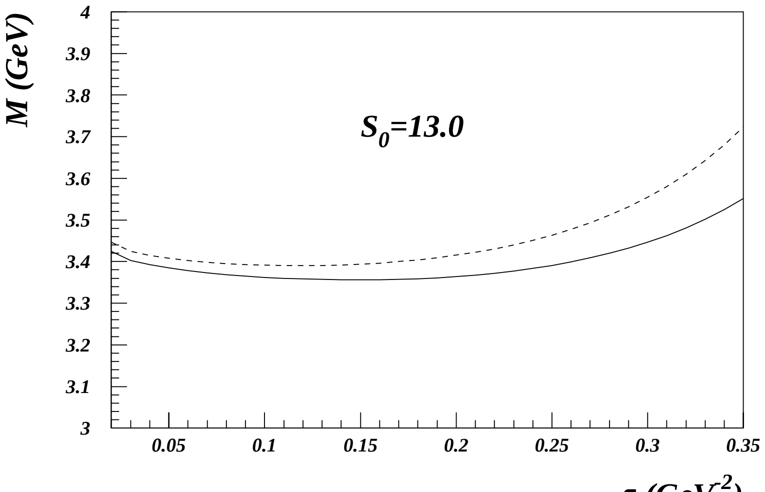


Fig. 3. 0^{++} hybrid mass from R_1/R_0 versus τ at $s_0 = 13.0\,\mathrm{GeV}^2$ corresponding to the current $j_\mu(x)$

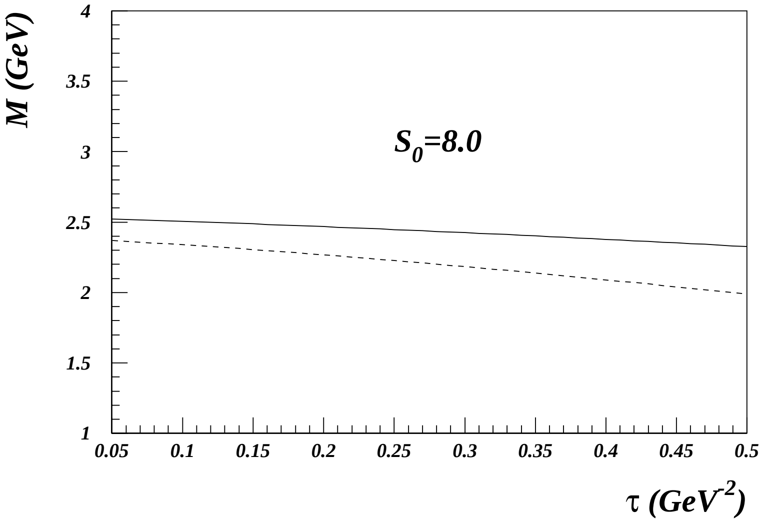


Fig. 4. 0^{-+} hybrid mass from R_1/R_0 versus τ at $s_0 = 8.0\,\mathrm{GeV}^2$ corresponding to the currents $j(x)$ and $j_\mu(x)$

be $8.0\,\mathrm{GeV}^2$, while the s_0 for the current $j_\mu(x)$ is chosen to be $13.0\,\mathrm{GeV}^2$. The results change slightly with s_0.

It is apparent that the mass of the light-quark hybrid meson depends on what interpolated current we choose: the mass of the 0^{++} hybrid from the current $j_\mu(x)$ is about 1.0 GeV higher than that from the current $j(x)$, while the masses of the 0^{-+} hybrid from the two different currents are approximately the same.

We know that the hybrid meson is a three-body system and the valence quark, anti-quark and gluon may have different internal J^{pc} combinations. J^{pc} of $\bar{q}q$ shows what kind of interaction occurs between $\bar{q}q$ and gluon. In fact, the interaction between quarks and gluon in the current $j(x)$ is in the magnetic form, so we can think that the J^{pc} of the combination $\bar{q}q$ in the current $j(x)$ is mainly 1^{+-}, while the interaction in the current $j_\mu(x)$ is in the electric form and the J^{pc} of $\bar{q}q$ is 1^{--}. Only the state with the same overall and 'local' quantum numbers can dominate the corresponding correlation function, where we refer to the quantum numbers of the intrinsic $\bar{q}q$ combination or gluon, such as J^{pc}, as the 'local' quantum numbers. Therefore, the correlation function which consists of the current $j(x)$ is dominated by the 0^{++} state with the gluon in the TE(1^{+-}) mode and the correlation function which consists of the current $j_\mu(x)$ is dominated by the 0^{++} state with the gluon in the TM(1^{--}) mode. The predicted 0^{++} masses from the different currents, correspondingly, are different. Let us look at diagram (c) in Fig. 1; the $\bar{\psi}\sigma^{\mu\nu}\psi$ breaks helicity conservation, so in the center-of-mass frame (also center-of-mass frame of the $\bar{q}$–q's), the total spin of the quark–antiquark is zero if the quarks are massless. On the other hand, the magnetic interaction dominates in the current $j(x)$. Therefore, the two-gluonic condensate term in the correlation function of $j(x)$ is proportional to the quark's mass (this is not the case for the current $j_\mu(x)$, see (3) and (11) When m goes to zero, this term yields a large gap between the masses predicted from $j(x)$ and $j_\mu(x)$. However, in the heavy hybrid system, we have $m^2 \sim Q^2$. This is the reason that the authors of [7] got almost the same masses from the two different sum rules for the heavy hybrid

It is helpful to note that the J^{pc} of the $\bar{q}q$ combination in the bag model has the same structure as that in the currents $j_\mu(x)$ and $j(x)$; thus, we can compare our picture with that in the bag model. In the bag model, the energy of the hybrid meson consists of the volume energy, the zero-point energy, the mode energy and the $O(\alpha_s)$ quantum corrections. The valence quarks and gluon in the $\bar{q}qg$ hybrid mesons may have different excited modes and each mode has a different energy. Besides, the $O(\alpha_s)$ quantum corrections are spin dependent, and they have a different energy corresponding to the different internal J^{pc} combinations of $\bar{q}q$ with the gluon. The quarks and gluon in the 0^{++} hybrid mesons may be in the $s_{\frac{1}{2}}s_{\frac{1}{2}}$TM mode with internal J^{pc}: $1^{--}\otimes 1^{--}$ or in the $s_{\frac{1}{2}}p_{\frac{1}{2}}$TE mode with internal J^{pc}: $1^{+-}\otimes 1^{+-}$, so the same overall J^{pc} states may have different energies.

4 Mixing of the 0^{++} hybrid meson with the glueball

In this section, we discuss the mixing effect [13] on the mass of the 0^{++} hybrid meson. Since the mass of the 0^{++} hybrid (3.4 GeV) from the current j_μ is much larger than the pure 0^{++} glueball (1.7 GeV) in the sum rules, we do not discuss this situation. Only the mixing between the 0^{++} hybrid (2.3 GeV) from the current $j(x)$ and the 0^{++} glueball is considered. We choose the scalar gluonic current

$$j_1(x) = \alpha_s G^a_{\mu\nu} G^a_{\mu\nu}(x) \tag{14}$$

for the 0^{++} glueball, and the current

$$j_2(x) = g\bar{q}\sigma_{\mu\alpha} G^a_{\alpha\mu} T^a q(x) \tag{15}$$

for the 0^{++} hybrid meson.

The correlation function of the current in (14) was given in [14]:

$$\begin{aligned}\Pi_1(q^2) = {} & a_0 (Q^2)^2 \ln(Q^2/\nu^2) + b_0\langle\alpha_s G^2\rangle \\ & + c_0 \frac{\langle gG^3\rangle}{Q^2} + d_0 \frac{\langle \alpha_s^2 G^4\rangle}{(Q^2)^2},\end{aligned} \tag{16}$$

where $Q^2 = -q^2 > 0$, and

$$\begin{aligned}a_0 &= -2(\tfrac{\alpha_s}{\pi})^2(1+\tfrac{51}{4}\tfrac{\alpha_s}{\pi}),\\ c_0 &= 8\alpha_s^2,\\ b_0 &= 4\alpha_s(1+\tfrac{49}{12}\tfrac{\alpha_s}{\pi}),\\ d_0 &= 8\pi\alpha_s,\\ \langle\alpha_s G^2\rangle &= \langle \alpha_s G^a_{\mu\nu} G^a_{\mu\nu}\rangle,\\ \langle gG^3\rangle &= \langle g f_{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}\rangle,\\ \langle\alpha_s^2 G^4\rangle &= 14\langle(\alpha_s f_{abc} G^a_{\mu\rho} G^b_{\rho\nu})^2\rangle - \langle(\alpha_s f_{abc} G^a_{\mu\rho} G^b_{\lambda\nu})^2\rangle.\end{aligned}$$

From (6), (16) and (3), we have the following expressions:

$$\begin{aligned}R_0(\tau,s_0) &= -\tfrac{2a_0}{\tau^3}[1-\rho_2(s_0\tau)] + c_0\langle gG^3\rangle + d_0\langle\alpha_s^2 G^4\rangle\tau,\\ R_1(\tau,s_0) &= -\tfrac{6a_0}{\tau^4}[1-\rho_3(s_0\tau)] - d_0\langle\alpha_s^2 G^4\rangle,\\ R_2(\tau,s_0) &= -\tfrac{24a_0}{\tau^5}[1-\rho_4(s_0\tau)],\\ R'_0(\tau,s_0) &= \tfrac{1}{\tau^4}\{6A[1-\rho_3(s_0\tau)] + B\tau^2[1-\rho_1(s_0\tau)]\\ &\quad + C\tau^3[1-\rho_0(s_0\tau)] + D\tau^4\},\\ R'_1(\tau,s_0) &= \tfrac{1}{\tau^5}\{24A[1-\rho_4(s_0\tau)] + 2B\tau^2[1-\rho_2(s_0\tau)]\\ &\quad + C\tau^3[1-\rho_1(s_0\tau)]\},\end{aligned} \tag{17}$$

where R_k and R'_k in (17) are the moments corresponding to the currents $j_1(x)$ and $j_2(x)$, respectively.

By using the low-energy theorem [11], we can construct another correlator with $j_1(x)$ and $j_2(x)$:

$$\begin{aligned}&\lim_{q\to 0} \mathrm{i}\int \mathrm{d}x\, \mathrm{e}^{\mathrm{i}qx}\langle 0|T[g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a q(x), \alpha_s G^2(0)]|0\rangle = \\ &\frac{40\pi}{9}\langle 0|g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a q|0\rangle.\end{aligned} \tag{18}$$

For the light quark, $\langle 0|g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a q|0\rangle$ can be expressed in terms of $\langle 0|\bar{q}q|0\rangle$ as [15]

$$\langle 0|g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a q|0\rangle = -m_0^2\langle 0|\bar{q}q|0\rangle, \tag{19}$$

where $m_0^2 \approx 0.8\,\mathrm{GeV}^2$.

In order to factorize the spectral density, the couplings of the currents to the physical states are defined in the following way:

$$\begin{aligned}\langle 0|j_1|H\rangle &= f_{12}m_2,\ \ \langle 0|j_1|G\rangle = f_{11}m_1,\\ \langle 0|j_2|H\rangle &= f_{22}m_2,\ \ \langle 0|j_2|G\rangle = f_{21}m_1,\end{aligned} \tag{20}$$

where m_1 and m_2 refer to the glueball mass (including a small part of the quark component) and the $\bar{q}qg$ hybrid meson mass (including a small part of the pure gluon component); $|H\rangle$ and $|G\rangle$ refer to the $\bar{q}qg$ hybrid-meson state and the glueball state, respectively. After choosing the approximation of two resonances plus a continuum state, the spectral density of the currents $j_1(x)$ and $j_2(x)$ read, respectively,

$$\begin{aligned}\mathrm{Im}\,\Pi_1(s) = {} & m_2^2 f_{12}^2\delta(s-m_2^2) + m_1^2 f_{11}^2\delta(s-m_1^2)\\ & + \frac{2}{\pi}s^2\alpha_s^2\theta(s-s_0),\end{aligned} \tag{21}$$

$$\begin{aligned}\mathrm{Im}\,\Pi_2(s) = {} & m_2^2 f_{22}^2\delta(s-m_2^2) + m_1^2 f_{21}^2\delta(s-m_1^2)\\ & + \pi(As^3+Bs+C)\theta(s-s_0).\end{aligned} \tag{22}$$

Then it is straightforward to get the moments:

$$R_0 = \frac{1}{\pi}(m_2^2\mathrm{e}^{-m_2^2\tau}f_{12}^2 + m_1^2\mathrm{e}^{-m_1^2\tau}f_{11}^2), \tag{23}$$

$$R_1 = \frac{1}{\pi}(m_2^4\mathrm{e}^{-m_2^2\tau}f_{12}^2 + m_1^4\mathrm{e}^{-m_1^2\tau}f_{11}^2), \tag{24}$$

$$R_2 = \frac{1}{\pi}(m_2^6\mathrm{e}^{-m_2^2\tau}f_{12}^2 + m_1^6\mathrm{e}^{-m_1^2\tau}f_{11}^2), \tag{25}$$

$$R'_0 = \frac{1}{\pi}(m_2^2\mathrm{e}^{-m_2^2\tau}f_{22}^2 + m_1^2\mathrm{e}^{-m_1^2\tau}f_{21}^2), \tag{26}$$

$$R'_1 = \frac{1}{\pi}(m_2^4\mathrm{e}^{-m_2^2\tau}f_{22}^2 + m_1^4\mathrm{e}^{-m_1^2\tau}f_{21}^2). \tag{27}$$

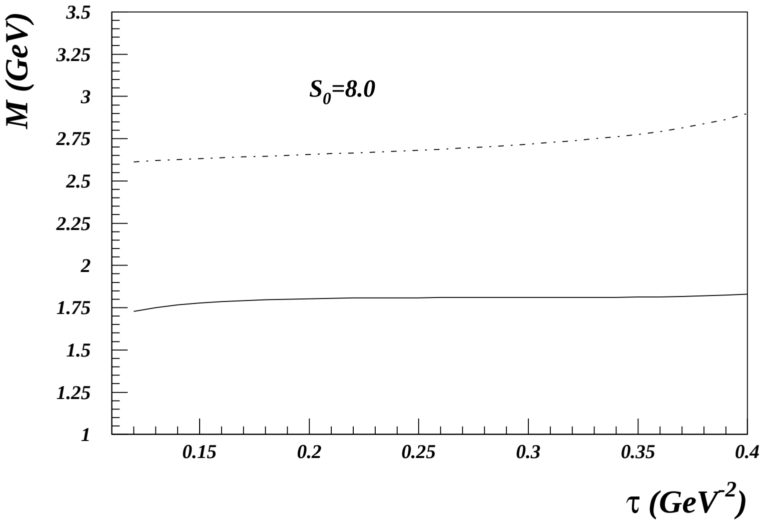


Fig. 5. 0^{++} $\bar{s}sg$ mass versus τ at $s_0 = 8.0\,\mathrm{GeV}^2$ corresponding to the mixing figure

In the meantime, assuming that the states $|G\rangle$ and $|H\rangle$ satisfy the l.h.s. of (18), one can obtain

$$\lim_{q\to 0} \mathrm{i} \int \mathrm{d}x\, \mathrm{e}^{\mathrm{i}qx} \langle 0|T[g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a q(x), \alpha_\mathrm{s}G^2(0)]|0\rangle = f_{22}f_{12} + f_{21}f_{11}. \tag{28}$$

To get the numerical result, the following additional parameters are chosen:

$$\langle gG^3\rangle = (0.27\,\mathrm{Ge}V^2)\langle\alpha_\mathrm{s}G^2\rangle,$$
$$\langle\alpha_\mathrm{s}^2G^4\rangle = \frac{9}{16}\langle\alpha_\mathrm{s}G^2\rangle^2,$$

The next step is to equate the QCD side with the hadron side term by term, and we get a set of equations. Given a reasonable range of the parameters s_0 and τ, a series of masses of the two states are obtained by solving this set of equations. Our result is illustrated in Fig. 5. The dotted line corresponds to the hybrid meson and the solid line corresponds to the glueball in this figure. It is shown that $s_0 = 8.0\,\mathrm{GeV}^2$ is the most favorable value. Then the prediction of the masses follows from the figure: the hybrid meson has a mass around 2.6 GeV and the glueball has a mass around 1.8 GeV. We conclude that the mixing makes the masses of the glueball and the hybrid meson both a little higher than those of their pure states.

5 Summary

In this paper, we calculate the 0^{++} and 0^{-+} masses of the light-quark hybrid mesons by using the QCD sum rules with two different kinds of interpolated currents: $g\bar{q}\sigma_{\mu\nu}G^a_{\nu\mu}T^a q(x)$ and $g\bar{q}\gamma_\alpha G^a_{\alpha\mu}T^a q(x)$. Our numerical result shows that the 0^{++} hybrid-meson mass from the current $j_\mu(x)$ is around 3.4 GeV, which is 1.0 GeV higher than that obtained from the current $j(x)$ (around 2.35 GeV). The masses of the 0^{-+} hybrid mesons from these two currents are approximately equal: 2.3 GeV.

The J^{pc} of the $\bar{q}q$ combination in these two currents are 1^{+-} and 1^{--}, respectively, so the interaction between the quarks and gluon is different and the two different kinds of 0^{++} or 0^{-+} hybrid mesons dominating the spectral density of these two different currents are different states accordingly. For the light-quark hybrids, the interaction between the quarks and the gluon makes a large contribution to the energy of the states; their masses may thus be different, while for the heavy-quark hybrid $m^2 \sim Q^2$, so different currents result in an approximately equal-mass prediction. Compared to the MIT bag model, our picture confirms that the mode analysis in the bag model is reasonable.

For the 0^{++} hybrid, the contribution of the two-gluon condensate to the correlation function (11) from the current $j_\mu(x)$ is large, while the contribution of the two-gluon condensate to the correlation function (3) from the current $j(x)$ is small because of the factor m^2 in the coefficient C; these two 0^{++} states have different mass values. For the 0^{-+} hybrid, the sign of the two-gluon condensate terms in (11) and (3) changes. This change slightly affects the correlation function and the mass prediction from the current $j(x)$ since the two-gluon condensate contribution to it is small. However, in the case of the current $j_\mu(x)$ the two-gluon condensate contribution to the correlation function is large. The change of sign causes the correlation function to differ much from the 0^{++} hybrid meson and this results in a much lower mass prediction for the 0^{-+} hybrid. These two 0^{-+} light hybrids obtained from the two different currents thus have approximately equal masses.

The mixing effect on the mass determination of the 0^{++} hybrid meson is considered too. We find that the mixing of the 0^{++} hybrid meson with the glueball shifts the masses of both the hybrid and the glueball a little up from their pure states.

Acknowledgements. This work is supported in part by the national Natural Science Foundation of P. R. China. H. Y. Jin thanks the Theoretical Physics Division at Fermilab for their kind hospitality.

References

1. D.R. Thompson et al.. (E852 Collab.), Phys. Rev. Lett. **79**, 1630 (1997); A. Abele et al.. (Crystal Barrel Collab.), Phys. Lett. B **423**, 175 (1998)
2. T. Barnes, F.E. Close, F. de Viron, Nucl. Phys. B **224**, 241 (1983); M. Chanowitz, S. Sharpe, Nucl. Phys. B **222**, 211 (1983)
3. J.M. Cronwall, S.F. Tuan, Phys. Lett. B **136**, 110 (1984)
4. N. Isgur, R. Kokoski, J. Paton, Phys. Rev. Lett. **54**, 869 (1985); F.E. Close, P.R. Page, Nucl. Phys. B **443**, 233 (1995); F.E. Close, P.R. Page, Phys. Rev. D **52**, 1706 (1995)
5. I.I. Balitsky, D.I. Dyakonov, A.V. Yung, Phys. Lett. B **112**, 71 (1982); I.I. Balitsky, D.I. Dyakonov, A.V. Yung, Sov. J. Nucl. Phys. **35**, 761 (1982); I.I. Balitsky, D.I. Dyakonov, A.V. Yung, Z. Phys. C **33**, 265 (1986)
6. J.I. Latorre, S. Narison, P. Pascual, R. Tarrach, Phys. Lett. B **147**, 169 (1984); J.I. Latorre, P. Pascual, S. Narison, Z. Phys. C **34**, 347 (1987)
7. J. Govaerts, F. de Viron, D. Gusbin, J. Weyers, Phys. Lett. B **128**, 262 (1983); J. Govaerts, F. de Viron, D. Gusbin, J. Weyers, Nucl. Phys. B **248**, 1 (1984); J. Govaerts, L.J. Reinders, P. Francken, X. Gonze, J. Weyers, Nucl. Phys. B **284**, 674 (1987)
8. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **147**, 385, 448 (1979)
9. F. de Viron, J. Govaerts, Phys. Rev. Lett. **44**, 2207 (1984); J.I. Latorre, P. Pascual, S. Narison, Z. Phys. C **34**, 347 (1987)
10. F. de Viron, J. Weyers, Nucl. Phys. B **185**, 391 (1981)
11. V.A. Novikov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **191**, 301 (1981)
12. V.A. Novikov, M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Fortschr. Phys. **32**, 585 (1984)
13. Tao Huang, H.Y. Jin, Ailin Zhang, Phys. Rev. D **59**, 034026 (1999)
14. E. Bagan, T.G. Steele, Phys. Lett. B **243**, 413 (1990)
15. L.J. Reinders, H. Rubinstein, Yazaki, Phys. Rep. **127**, 1 (1985)